

Calculus 1

Resit Exam

November 26, 2024 (18:30 – 20:30)



university of
 groningen

Please read the instructions!

1) Apply L'Hospital's Rule to evaluate the limit $\lim_{x \rightarrow 0} \frac{a^x - a^{\sin x}}{x^3}$ for all $a > 0$. Indicate the results (e.g. limit laws, continuity, differentiation rules) used in each step.

2) Use Taylor Series to find the limit $\lim_{x \rightarrow \infty} \left[x - x^2 \ln \left(1 + \frac{1}{x} \right) \right]$. Justify each step.

3) Use integration to calculate the total length of an astroid curve given by the equation $x^{2/3} + y^{2/3} = c^{2/3}$, where c is a positive constant.

4) Evaluate the definite integral $\int_1^2 \frac{x+2}{x\sqrt{4-x^2}} dx$.

5) Solve the initial value problem $y'(x) + xy(x) = x^3$, $y(0) = 1$.

6) Solve the following initial value problem

$$y''(x) + 2y'(x) + 2y(x) = 0, \quad y(0) = 1, \quad y'(0) = -3.$$